

$$\sin^5 x + \cos^5 x = 1 \text{ решить разложив по } a^5 + b^5 = \dots$$

$$f(a) = a^5 + b^5$$

$$a = -b$$

$$\text{делится на } (a - \text{корень}) = a - (-b) = a + b$$

$$a^5 + 0a^4 + 0a^3 + 0a^2 + 0a + b^5 | a + b$$

$$a^5 + a^4b \quad | a^4 - a^3b + a^2b^2 - ab^3 + b^4$$

$$-a^4b + 0a^3$$

$$-a^4b + -a^3b^2$$

$$a^3b^2 + 0a^2$$

$$a^3b^2 + a^2b^3$$

$$-a^2b^3 + 0a$$

$$-a^2b^3 - ab^4$$

$$ab^4 + b^5$$

$$ab^4 + b^5$$

$$0$$

$$a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$$

$$(\sin x + \cos x)(\sin^4 x - \sin^3 x \cos x + \sin^2 x \cos^2 x - \sin x \cos^3 x + \cos^4 x) = 1$$

$$(\sin x + \cos x)(\sin^4 x - \sin x \cos x + 2\sin^2 x \cos^2 x - \sin^2 x \cos^2 x + \cos^4 x) = 1$$

$$(\sin x + \cos x)(1 - \sin x \cos x - \sin^2 x \cos^2 x) = 1$$

ЗАМЕНА

$$\sin x + \cos x = t \Rightarrow \sin^2 x + 2\sin x \cos x + \cos^2 x = t^2 \Rightarrow \sin x \cos x = (t^2 - 1)/2$$

$$t(1 - (t^2 - 1)/2 - (t^2 - 1)^2/4) = 1$$

$$t - (t^3 - t)/2 - (t^3 - t)^2/4 = 1$$

$$4t - 2(t^3 - t) - (t^2 - 1)^2 = 4$$

$$4t - 2t^3 + 2t - t(t^4 - 2t^2 + 1) = 4$$

$$4t - 2t^3 + 2t - t^5 + 2t^3 - t - 4 = 0$$

$$-t^5 + 5t - 4 = 0$$

$$t^5 - 5t + 4 = 0$$

$$t^4 + t^3 + t^2 + t - 4 = 0$$

$$t = 1$$

$$t^3 + 2t^2 + 3t + 4 = 0$$

$$\text{корень меньше } y = -\sqrt{2} \quad \sin(x + \pi/4) = y/\sqrt{2} < -1 \text{ решений нет}$$

$$\sin x + \cos x = 1$$

$$\sqrt{2}\sin(x + \pi/4) = 1$$

$$\sin(x + \pi/4) = \sqrt{2}/2$$

$$x + \pi/4 = \pi/4 + 2\pi k$$

$$x = 2\pi k$$

$$x + \pi/4 = 3\pi/4 + 2\pi k$$

$$x = \pi/2 + 2\pi k$$

$$\sin^{2003} x + \cos^{2003} x = 1$$

$$\sin x = 0 \quad \cos x = 1$$

$$x = 2\pi k \quad x = 2\pi k$$

$$\sin x = 1 \quad \cos x = 0$$

$$x = \pi/2 + 2\pi k \quad x = \pi/2 + 2\pi k$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin x = (-1; 1) \setminus \{0\}$$

$$\cos x = (-1; 1) \setminus \{0\}$$

$$\sin^{2003} x + \cos^{2003} x < 1$$